

Du  
**18**  
NOV.  
2022

16h30

-  
17h30

## SÉMINAIRE BOURBAKI DU VENDREDI

**Aled Walker — Pointwise convergence and exponential sums**

IHP  
Darboux

### INSCRIPTION

The central questions of ergodic theory concern the convergence of various averages, taken over the orbits of a dynamical system. If the averages in question are taken over consecutive elements of an orbit, Birkhoff's foundational 'pointwise ergodic theorem' shows that they all converge (apart from at a zero measure set of orbits). Applications include the strong law of large numbers and certain properties of the continued fraction expansion of irrational numbers. In this introductory talk, we will sketch the classical 'density' proof of Birkhoff's result, before discussing the challenges of generalising this proof to the setting of sparser averages over the orbits — averages over higher degree polynomial values, say. In particular, we will expose a few of the ideas from Bourgain's hugely creative works of the late 1980s, which establish a link between questions of pointwise convergence and estimates in discrete harmonic analysis. Exponential sums enter the picture: we will analyse the simplest arithmetic examples, with a focus on Weyl's inequality for an exponential sum of monomials.



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### **HORAIRES**

L'institut :

- lundi au vendredi de 8h30 à 18h,
- fermé les jours fériés.

Le musée - Maison Poincaré :

- lundi, mardi, jeudi et vendredi de 9h30 à 17h30,
- samedi de 10h à 18h,
- fermé le mercredi et le dimanche.