



Du  
**13**  
MARS.  
2023

14h00  
-  
15h00

## RENCONTRES DE THÉORIE ANALYTIQUE DES NOMBRES

### Explicit constructions of multiplicative functions with small correlations

Salle Grisvard, IHP, Paris

#### INSCRIPTION

There is a well-known relationship between the distribution of primes and distribution of  $\pm 1$  signs of the Liouville function (the completely multiplicative function taking the value  $\pm 1$  at all primes). A conjecture of Chowla, analogising the Hardy-Littlewood prime  $k$ -tuples conjecture, predicts that the autocorrelations of  $\lambda$ , e.g.  $\frac{1}{x} \sum_{n \leq x} \lambda_{n+1} \cdots \lambda_{n+k}$  tend to 0 on average as  $x$  tends to  $\infty$ . This conjecture, along with its generalisation to the broader collection of bounded "non-pretentious" multiplicative functions, due originally to Elliott, remain wide open for  $k \geq 2$ . Previously, there were no explicit examples in the literature of (deterministic and scale-independent) non-pretentious multiplicative functions known to satisfy Elliott's conjecture. In this talk I will present a construction of a non-pretentious multiplicative function  $f : \mathbb{N} \rightarrow \{-1, 1\}$  all of whose auto-correlations tend to 0 on average, answering a (ergodic theory) question of Lemanczyk and de la Rue. I will further discuss the following applications of this construction:

- i) a proof that Chowla's conjecture does not imply the Riemann Hypothesis, i.e., there are  $\pm 1$ -valued multiplicative functions  $f$  all of whose autocorrelations tend to 0, but that do not exhibit square-root cancellation on average (the object of some recent speculation);
- ii) there are multiplicative subsemigroups of  $\mathbb{N}$  with Poissonian gap statistics, thus giving an unconditional multiplicative analogue of a classical result of Gallagher about primes in short intervals.

URL of the page: <https://www.ihp.fr/fr/agenda/explicit-constructions-multiplicative-functions-small-correlations>



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### **HORAIRES**

L'institut :

- lundi au vendredi de 8h30 à 18h,
- fermé les jours fériés.

Le musée - Maison Poincaré :

- lundi, mardi, jeudi et vendredi de 9h30 à 17h30,
- samedi de 10h à 18h,
- fermé le mercredi et le dimanche.