Du
13
MARS.
2023

14h00
-
15h00

RENCONTRES DE THÉORIE ANALYTIQUE DES NOMBRES

Explicit constructions of multiplicative functions with small correlations

Salle Grisvard, IHP, Paris

INSCRIPTION

There is a well-known relationship between the distribution of primes and distribution of ± 1 signs of the Liouville function (the completely multiplicative function taking the value -1 at all primes). A conjecture of Chowla, analogising the Hardy-Littlewood prime k-tuples conjecture, predicts that the autocorrelations of λ, e.g. $\frac{1}{x} \sum_{n \leq x} \lambda_{n+1} \cdots\lambda_{n+k}$ tend to 0 on average as x tends to ∞. This conjecture, along with its generalisation to the broader collection of bounded "non-pretentious" multiplicative functions, due originally to Elliott, remain wide open for $k \geq 2$. Previously, there were no explicit examples in the literature of (deterministic and scale-independent) non-pretentious multiplicative functions known to satisfy Elliott's conjecture. In this talk I will present a construction of a non-pretentious multiplicative function $f : \mathbf{N} \rightarrow \{-1,1\}$ all of whose auto-correlations tend to 0 on average, answering a(n ergodic theory) question of Lemanczyk and de la Rue. I will further discuss the following applications of this construction:

i) a proof that Chowla's conjecture does not imply the Riemann Hypothesis, i.e., there are ± 1-valued multiplicative functions f all of whose autocorrelations tend to 0, but that do not exhibit square-root cancellation on average (the object of some recent speculation);

ii) there are multiplicative subsemigroups of $\mathbf{N}$ with Poissonian gap statistics, thus giving an unconditional multiplicative analogue of a classical result of Gallagher about primes in short intervals.

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(Joint work with Oleksiy Klurman, Par Kurlberg and Joni Teravainen.)

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HORAIRES

L'institut :
• lundi au vendredi de 8h30 à 18h,
• fermé les jours fériés.

Le musée - Maison Poincaré :
• lundi, mardi, jeudi et vendredi
de 9h30 à 17h30,
• samedi de 10h à 18h,
• fermé le mercredi et le dimanche.

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