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01
AVR.
2023

10h00
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11h00

SÉMINAIRE BOURBAKI

**Jonathan Hickman — Pointwise convergence for the Schrödinger equation , after
Xiumin Du and Ruixiang Zhang**

IHP
Hermite

INSCRIPTION

For an initial datum $f \in L^2(\mathbb{R}^n)$, consider the linear Schrödinger equation

$$i\partial_t u - \Delta_x u = 0,$$

$$(x, t) \in \mathbb{R}^n \times \mathbb{R}.$$

$$u(x, 0) = f(x)$$

In 1980, Carleson asked which additional conditions on f guarantee

(*)

$$\lim_{t \rightarrow 0} u(x, t) = f(x)$$

for almost every $x \in \mathbb{R}^n$.

$t \rightarrow 0$

More precisely, what is the minimal Sobolev regularity index s such that (*) holds whenever $f \in H^s(\mathbb{R}^n)$?

Whilst the $n = 1$ case was fully understood by the early 1980s, in higher dimensions the situation is much more nuanced. Nevertheless, a recent series of dramatic developments brought about an almost complete resolution of the problem. First Bourgain 2016 produced a subtle counterexample demonstrating that pointwise convergence can fail for certain $f \in H^s(\mathbb{R}^n)$ with $s < n/(2n+2)$.

. Complementing

this, convergence was then shown to hold for $s > n/(2n+2)$

when $n = 2$ in a landmark

$2(n+1)$ paper of Du, Guth and Li 2017 and later in all dimensions in equally important work of Du and Zhang 2019.

This seminar will explore the positive result of Du and Zhang 2019. The argument combines sophisticated modern machinery from harmonic analysis such as the multilinear Strichartz estimates of Bennett, Carbery and Tao 2006 and the l^2 decoupling theory of Bourgain and Demeter 2015. However, equally important are a variety of elementary guiding principles, rooted in Fourier analysis, which govern the behaviour of solutions to the Schrödinger equation. The talk will focus on these basic Fourier analytic principles, building intuition and presenting a powerful toolbox for tackling problems in modern PDE and harmonic analysis.



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HORAIRES

L'institut :

- lundi au vendredi de 8h30 à 18h,
- fermé les jours fériés.

Le musée - Maison Poincaré :

- lundi, mardi, jeudi et vendredi de 9h30 à 17h30,
- samedi de 10h à 18h,
- fermé le mercredi et le dimanche.